Available online at www.sciencedirect.com
ScienceDirect
Computational Statistics and Data Analysis 53 (2009) 2168–2188
COMPUTATIONAL STATISTICS & DATA ANALYSIS
www.elsevier.com/locate/csda

The effects of misspecified marginals and copulas on computing the value at risk: A Monte Carlo study

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Available online 8 February 2008

Abstract

The effect on the estimation of the Value at Risk when dealing with multivariate portfolios when there is a misspecification both in the marginals and in the copulas is investigated. It is first shown that, when there is skewness in the data and symmetric marginals are used, the estimated elliptical (normal or t) copula correlations are negatively biased, reaching values as high as 70% of the true values. Besides, the bias almost doubles if negative correlations are considered, compared to positive correlations. As for the *t* copula degrees of freedom parameter, the use of wrong marginals delivers large positive biases, instead. If the dependence structure is represented by a copula which is not elliptical, e.g. the Clayton copula, the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of marginal skewness. Extensive Monte Carlo studies then show that the misspecifications in the marginal volatility equation more than offset the biases in copula parameters when VaR forecasting is of concern, small samples are considered and the data are leptokurtic. The biases in the volatility parameters are much smaller, whereas those ones in the copula parameters remain almost unchanged or even increase when the sample dimension increases. In this case, copula misspecifications do play a role for VaR estimation. However, these effects depend heavily on the sign of the dependence.

1. Introduction

Value at Risk (VaR) has become one of the most popular risk measures since it was recommended and adopted by the Bank of International Settlements and USA regulatory agencies in 1988. Jorion (2000) provides an introduction to Value at Risk as well as discussing its estimation, while the www.gloriamundi.org website comprehensively cites the Value at Risk literature as well as providing other VaR resources. Recently, Ané and Kharoubi (2003), Junker and May (2005) and Fantazzini (in press) investigated the influence of different bivariate joint distribution assumptions on the Value at Risk. They found that the best results can be achieved by models allowing for heavily tailed marginals and a modelling of tail dependency and asymmetric tail dependency. However, they also found out that, for short positions, a multivariate normal model with dynamic normal marginals and constant normal copula can be a proper choice.

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0167-9473/$ - see front matter
doi:10.1016/j.csda.2008.02.002

The aim of this paper is to investigate how misspecifications both in the marginals and in the copulas can affect the estimation of the Value at Risk when dealing with multivariate portfolios. Given the partitioning of the multivariate parameter vector into separate parameters for each margin and parameters for the copula, it has become usual practice to break up the optimization problem into several small optimizations, each with fewer parameters. This multi-step procedure is known as the method of Inference Functions for Margins (IFM); e.g., see Patton (2004), McNeil et al. (2005), Patton (2006a), Cherubini et al. (2004), Gunky et al. (2007) and Karlis and Nikoloulopoulos (2008). Therefore, in order to achieve our aim, we first analyze the effects of misspecified marginals on the multivariate parameter vector estimation, while we examine in a second step the effects of model misspecification on the computation of the Value at Risk. Regarding the first point, we find the interesting result that, when there is skewness in the data and symmetric marginals are used, the estimated normal copula correlations are negatively biased, and the bias increases when moving from the Student's t to the normal distribution, reaching values as high as 27% of the true values. We find that the bias almost doubles if negative correlations are considered, compared to positive correlations. When the true dependence function is represented by the t copula, the choice of the marginals tends to have much stronger effects on copula parameter estimation, with biases up to 50% of the true values for the correlations and up to 380% for the t copula degrees of freedom parameter. If the dependence structure is represented by a copula which is not elliptical, e.g. the Clayton copula, the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of the marginal skewness.

The second contribution of the paper is an extensive Monte Carlo study carried out to assess the potential impact of both misspecified margins and misspecified copulas on the estimation of multivariate VaR for equally weighted portfolios. We find that, when small samples are considered and the data are leptokurtic, the overestimation/underestimation in the marginal volatility parameters is so large as to offset the negative biases in copula parameters, thus delivering very conservative VaR estimates for all quantiles. This is true for all multivariate models considered in the analysis.

When the sample dimension increases, the biases in the volatility parameters are much smaller, whereas those in the copula parameters remain almost unchanged or even increase, like for the t copula and the Clayton copula parameters, when (wrong) symmetric marginals are used instead of skewed ones. In this case, copula misspecifications do play a role. However, these effects depend heavily on the sign of the dependence: if it is negative, the bias can be as large as 70%, like for t copula correlations, for instance. If it is positive, the bias is much smaller (10% or less for the t copula correlations), and the effects on quantile estimation are much more limited, if not completely offset by marginal misspecifications. Therefore, this Monte Carlo evidence gives some insights into why previous empirical literature found that the influence of a misspecification in the copula is given with 20% or less of the whole estimation error for the VaR; see e.g. Ané and Kharoubi (2003) and Junker and May (2005).

Finally, we perform an empirical analysis with ten trivariate portfolios, where we quantify the risk of the portfolio under different joint distribution assumptions. The rest of the paper is organized as follows. Section 2 presents the copula–VAR–GARCH models, while in Section 3 we perform simulation studies to assess the finite sample properties of these models under different DGPs. In Section 4, we conduct an empirical analysis with ten portfolios composed of stocks quoted at the NYSE, while we conclude in Section 5.

2. Copula–VAR–GARCH modelling

Consider a general copula–vector autoregression model, where the n endogenous variables $x_{i,t}$ are explained by an intercept μ_i, autoregressive terms of order p, and an error term $\sqrt{h_{i,t}}\eta_{i,t}$:

$$\begin{aligned} x_{1,t} &= \mu_1 + \sum_{i=1,}^{n}\sum_{l=1}^{p}\phi_{1,i,l}x_{i,t-l} + \sqrt{h_{1,t}}\,\eta_{1,t} \\ &\vdots \\ x_{n,t} &= \mu_n + \sum_{i=1,}^{n}\sum_{l=1}^{p}\phi_{n,i,l}x_{i,t-l} + \sqrt{h_{n,t}}\eta_{n,t}. \end{aligned} \tag{1}$$

Let the standardized innovations $\eta_{i,t}$ have mean zero and variance one, while $\sqrt{h_{i,t}}$ can be constant or time varying like in GARCH(1, 1) models; see Bollerslev et al. (1994) for a detailed survey of GARCH models:

$$\begin{aligned} h_{1,t} &= \omega_1 + \alpha_1 \left(\eta_{1,t-1}\sqrt{h_{1,t-1}}\right)^2 + \beta_1 h_{1,t-1} \\ &\vdots \\ h_{n,t} &= \omega_n + \alpha_n \left(\eta_{n,t-1}\sqrt{h_{n,t-1}}\right)^2 + \beta_n h_{n,t-1}. \end{aligned} \tag{2}$$

Furthermore, the $\eta_{i,t}$ have a conditional joint distribution $H_t\left(\eta_{1,t}, \dots, \eta_{n,t}; \theta\right)$ with parameters vector θ. Copula theory provides an easy way to deal with the (otherwise) complex multivariate modelling. The essential idea of the copula approach is that a joint distribution can be factored into the marginals and a dependence function called a copula. The joint distribution $H_t\left(\eta_{1,t}, \dots, \eta_{n,t}; \theta\right)$ can be expressed as follows, thanks to the so-called Sklar theorem (1959):

$$\left(\eta_{1,t}, \dots, \eta_{n,t}\right) \sim H_t\left(\eta_{1,t}, \dots, \eta_{n,t}; \boldsymbol{\theta}\right) = C_t(F_{1,t}(\eta_{1,t}; \delta_1), \dots, F_{n,t}(\eta_{n,t}; \delta_n); \boldsymbol{\gamma}), \tag{3}$$

that is the joint distribution H_t of a vector of innovations $\eta_{i,t}$ is the copula $C_t\left(\cdot\,; \gamma\right)$ of the cumulative distribution functions of the innovation marginals $F_{1,t}(\eta_{1,t}; \delta_1), \dots, F_{n,t}(\eta_{n,t}; \delta_n)$, where $\gamma, \delta_1, \dots, \delta_n$ are the copula and marginal parameters, respectively. The study of copulas originated with the seminal papers by Sklar (1959) and has seen various applications in the statistics and financial literature. For more details, we refer the interested reader to the recent methodological overviews by Joe (1997) and Nelsen (1999), while Cherubini et al. (2004) provide a detailed discussion of copula techniques for financial applications. It follows from (3) that the log-likelihood function for the joint conditional distribution $H_t(\,\cdot\,; \theta)$ is given by

$$l(\theta) = \sum_{t=1}^{T} \log(c(F_1(x_{1,t}; \delta_1), \dots, F_n(x_{n,t}; \delta_n); \boldsymbol{\gamma})) + \sum_{t=1}^{T}\sum_{i=1}^{n} \log f_i(x_{i,t}; \delta_{i,t}), \tag{4}$$

where c is the copula density function, whereas f_i are the marginal densities. Hence, the log likelihood of the joint distribution is just the sum of the log likelihoods of the margins and the log likelihood of the copula. Standard ML estimates may be obtained by maximizing the above expression with respect to the parameters $(\delta_1, \dots, \delta_n; \gamma)$. In practice this can involve a large numerical optimization problem with many parameters which can be difficult to solve. However, given the partitioning of the parameter vector into separate parameters for each margin and parameters for the copula, one may use (4) to break up the optimization problem into several small optimizations, each with fewer parameters. This multi-step procedure is known as the method of Inference Functions for Margins (*IFM*); see Joe and Xu (1996) and Joe (1997). Joe (1997) compares the efficiency of the IFM method relative to full maximum likelihood for a number of multivariate models and finds the IFM method to be highly efficient. Therefore, we think it safe to use the IFM method and benefit from the huge reduction in complexity that it implies for the numerical optimization.

3. Simulation studies

In this section we present the results of the simulation studies concerning the following specification for a trivariate copula–VAR(1)–GARCH(1, 1) model:

$$Y_t = \mu + \Phi_Y^{(1)} Y_{t-1} + \sqrt{h_t}\eta_t, \tag{5}$$

where the matrix h is diagonal and contains the variances:

$$h_{i,t} = \omega_i + \alpha_i(\eta_{i,t-1}\sqrt{h_{i,t-1}})^2 + \beta_i h_{i,t-1}, \quad i = 1, 2, 3,$$

and where we consider the following possible Data Generating Processes (DGPs):

(1) We examine six different types of marginals for the innovations η_t:
- A Generalized t (see Hansen (1994)) with skewness parameter $\lambda = -0.5$ and degrees of freedom $\nu = 3$.

- A Generalized t with skewness parameter $\lambda = 0$ and degrees of freedom $\nu = 3$, that is a symmetric Student's t distribution.
- A Generalized t with skewness parameter $\lambda = 0.5$ and degrees of freedom $\nu = 3$.
- A Generalized t with skewness parameter $\lambda = -0.5$ and degrees of freedom $\nu = 10$.
- A Generalized t with skewness parameter $\lambda = 0$ and degrees of freedom $\nu = 10$, that is a symmetric Student's t distribution close to a standard normal distribution.
- A Generalized t with skewness parameter $\lambda = 0.5$ and degrees of freedom $\nu = 10$.

(2) We examine three types of copulas to model the joint dependence (3) of the innovations η_t:
- A normal copula with correlation matrix
$$\Sigma = \begin{pmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{pmatrix}.$$
- A Student's t copula with correlation matrix
$$\Sigma = \begin{pmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{pmatrix},$$
and degrees of freedom $\nu_c = 3$.
- A Clayton copula with parameter $\alpha_c = 3$.

We chose the normal and t copulas given their widespread use in financial applications (see e.g. Cherubini et al. (2004) and McNeil et al. (2005)), whereas we chose the Clayton copula because it has positive lower tail dependence and a simulation algorithm that can be easily implemented even for highly dimensional portfolios, unlike all other Archimedean copulas. We do not consider time-varying copulas in this work to keep the number of simulated DGPs tractable. Besides, recent literature (see Chen et al. (2004) and Patton (2004)) has shown that a simple normal copula with no dynamics is sufficient for describing the joint dependence structure for financial data in most cases. Only when the number of variables considered is higher than 20 do statistically significant differences start to emerge and the (constant) Student's t copula become required. Fantazzini et al. (in press) found similar evidence with monthly operational risk data, too.

(3) We consider two possible data situations: $T = 500$, $T = 2000$.

(4) We consider the same conditional mean and variance specification for all the three variables:
$$\mu = \begin{bmatrix} 0.15 \\ 0.15 \\ 0.15 \end{bmatrix}, \qquad \Phi_Y^{(1)} = \begin{bmatrix} 0.5 & -0.2 & 0.15 \\ 0.5 & -0.2 & 0.15 \\ 0.5 & -0.2 & 0.15 \end{bmatrix}, \qquad \omega = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix},$$
$$\alpha = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}, \qquad \beta = \begin{bmatrix} 0.9 \\ 0.9 \\ 0.9 \end{bmatrix}.$$

We justify this choice as made for the sake of simplicity and again to keep the number of simulated DGPs tractable. However, we chose values to mimic the most common stylized facts of financial markets, such as strong persistence in conditional variances; see, for example, Tsay (2002) and references therein. We also tried other specifications for $\Phi_Y^{(1)}$, but the simulation results were robust with respect to this choice.

We generated 1000 Monte Carlo samples for each marginal and copula specification previously described and we then estimated the nine models reported in Table 1. We first analyze in the next two subsections the effects of misspecified marginals on marginal and copula parameter estimation, since they will later help us explain the effects of model misspecification on the computation of the Value at Risk.

3.1. Effects of marginal misspecifications on marginal parameter estimation

We report here only the main findings, while the full set of results are reported in the technical report by Fantazzini (2007) in Tables 3–8. The simulation studies show some interesting results:

- ***Normal* [Model (1)–Model (4)–Model (7)]**:
  . *Conditional mean*: The estimated parameters do not show any particular bias or trend.

Table 1
Multivariate distribution specifications

MODEL	**Marg.**	**Copula**	MODEL	**Marg.s**	**Copula**	MODEL	**Marg.**	**Copula**
Model (1)	Normal	Normal	**Model (4)**	Normal	t copula	**Model (7)**	Normal	Clayton
Model (2)	St. t	Normal	**Model (5)**	St. t	t copula	**Model (8)**	St. t	Clayton
Model (3)	Gen. t	Normal	**Model (6)**	Gen. t	t copula	**Model (9)**	Gen. t	Clayton

. *Conditional variance*: The lack of a parameter modelling the fat tails induces a large positive bias in the parameters ω and negative ones in α and β. The mean biases of these parameters can be larger than 110%, 50% and 10%, respectively, when $T = 500$, $\nu = 3$ and the data are skewed, while they are slightly smaller when the data are symmetric. Instead, if we consider the median estimates, the biases are much smaller: these differences will later explain why mean and median VaR estimates are very different when small samples are considered. These results indicate the difficulties of estimating GARCH models with small samples, thus confirming previous simulation evidence from Hwang and Valls Pereira (2006) which, however, considered only univariate models with normally distributed errors and did not examine the effect of different joint distributions. Moreover, the strong biases were expected, since it has been shown by Newey and Steigerwald (1997) in a theoretical work that a QML estimator can be biased when data are not symmetric.
. *Sample dimension*: When the dimension T increases, the biases and the RMSE decrease for all parameters.
. *Copula dependence structure*: No qualitative differences are found across different types of dependence functions.

- ***Student's t*** **[Model (2)–Model (5)–Model (8)]**:

. *Conditional mean*: If the true DGP has skewed marginals but we use the Student's t, the biases in the conditional mean parameters can be as high as 45% of the true values for the parameter μ, when $T = 500$ and $\nu = 3$. Instead, no differences are found between left skewed and right skewed data.
. *Conditional variance*: The use of this symmetric distribution causes severe biases in the ω and α estimates, when $T = 500$ and $\nu = 3$ and the data are skewed: up to 90% and 25% of the true values for the former two parameters, respectively. Instead, the parameter β is estimated much more precisely, with biases smaller than 7%. However, like for the normal distribution, the median estimates are more precise: up to 30% and 8% for the ω and α, whereas the negative bias for β decreases to 2%.
. *Degrees of freedom*: When the data are skewed, there is a negative bias that increases both with the sample dimension and the magnitude of ν. It can be higher than 25% of the true value when $T = 2000$.
. *Sample dimension*: Except for the degrees of freedom previously discussed, when the dimension T increases, the biases and the RMSE decrease as well.
. *Copula dependence structure*: No qualitative differences are found across different types of dependence functions.

- ***Generalized t*** **[Model (3)–Model (6)–Model (9)]**:

. *Conditional mean*: The estimated parameters of the conditional mean do not show any particular bias or trend.
. *Conditional variance*: When there is skewness in the data, the estimated parameters are estimated more precisely than when the observations are symmetric: in the latter case, the constant ω shows a strong positive bias which can be higher than 20% (median) when $T = 500$, while in the former case the bias is around 10% (median). These differences are even larger if mean estimates are considered. This difference can be explained considering that the Generalized t is not the most efficient model when the data are symmetric. However, these positive biases are present for all the distributions considered, and clearly highlight how difficult estimating a GARCH model for small samples is.
. *Degrees of freedom*: As expected, the degrees of freedom ν are estimated much more precisely when they are low than vice versa (positive mean bias around 35% when $T = 500$), given that this parameter is much more difficult to identify when it is high in magnitude. If the sample dimension increases, the biases and the RMSE decrease as expected.
. *Skewness parameter*: The estimates show a negative mean bias around 15% for small samples with fat tails, i.e. $\nu = 3$, which disappears as the sample dimension increases. Small samples and thinner tails, i.e. $\nu = 10$, mitigate the bias to 10%. Similar but smaller biases hold when the median is considered.
. *Sample dimension*: When the dimension T increases, the biases and the RMSE decrease for all parameters.

. *Copula dependence structure*: No difference is found across different types of dependence functions: this result was expected since the Generalized t is the true marginal specification and, in this case, its parameters are variation free with respect to the copula parameters (see Patton (2006a,b)).

3.2. Effects of marginal misspecifications on copula parameter estimation

The full set of results for this part are reported in the technical report by Fantazzini (2007) in Tables 9–14. The main findings can be summarized as follows:

- ***Normal copula***:
  . *Correlation matrix*: When there is skewness in the data and symmetric marginals are used, the estimated correlations are negatively biased, and the bias increases when moving from the Student's t to the normal distribution, reaching values as high as 27% of the true values. It is interesting to note that the bias almost doubles if negative correlations (-0.5) are considered, compared to the positive correlation case. This result remains the same both with positively skewed and with negatively skewed data, while no biases are found if the variables are uncorrelated. If the true Generalized t is used, the estimation biases are equal to or smaller than 1% even with $T = 500$. No differences between the mean and the median estimates are found. When the degrees of freedom of the true marginals increase from 3 to 10, the biases in the correlations decrease as expected, but the previous conclusions remain unchanged: the biases for negative correlations are double compared to those for positive correlations, and increase when the used marginals are normally distributed.
  . *Sample dimension*: No major differences in the biases are found when moving from $T = 500$ to $T = 2000$, even though the RMSE are smaller.
- ***T copula***:
  . *Correlation matrix*: When the true dependence function is represented by the t copula, the choice of the marginals tends to have much stronger effects on copula parameter estimation. If the true DGP has skewed marginals but we use symmetric ones, instead, the biases in the correlations can be as high as 15% of the true values for the Student's t and 50% when using the normal distribution and $T = 500$. The difference between negative (-0.5) and positive (0.5) correlations is even more striking here: the bias in the former case is, in general, five times the bias in the latter case. Interestingly, if the true correlations are negative and normal marginals are used, then the bias tends to increase with the sample dimension, while this effect is weaker or negligible with positive correlations. Instead, if Student's t distributions are used, the biases tend to decrease in all cases as the sample dimension increases.

  Differently from what we saw with the normal copula, the use of normal marginals when the true ones are skewed induces positive correlation between variables that are uncorrelated. This effect is much smaller if Student's t marginals are used, instead. Furthermore, even if we use the true Generalized t, nevertheless the correlations can be negatively biased in small samples.

  Similarly to the case for the normal copula, when the degrees of freedom of the true marginals increase from 3 to 10, the biases in the correlations decrease, but the previous qualitative conclusions remain unchanged. However, we also note that the biases tend to remain as high as 50% when using the normal distribution for the marginals, if negative correlations are of concern.
  . *t copula degrees of freedom* ν_c: The use of wrong marginals can have dramatic effects, with positive biases ranging from 60% for the Student's t distribution up to 380% for the normal distribution, when the true DGPs are skewed and $T = 500$. Besides, we note that the estimates can be poor for small samples also when using the Generalized t, with biases around 5% of the true values. Instead, when the sample dimension increases and the true marginal degrees of freedom increase to 10, the estimation bias decreases in all cases. However, it still remains as high as 40% for the symmetric Student's t and over 100% for the normal distributions.
- ***Clayton copula***: This copula is not elliptical and the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of marginal skewness.
  . If the *true marginals show negative skewness*, then using symmetric marginals causes the Clayton parameter α_c to be negatively biased, where the larger bias is caused by the normal distribution. Increasing the sample dimension to $T = 2000$ slightly decreases these biases, but they still remain higher than 20% for the normal distribution and 17% for the symmetric student's t. Remarkably, if we increase the marginal degrees of freedom of the true DGPs to 10, the biases still remain higher than 15% for all the symmetric distributions.

. If the *true marginals are symmetric*, the estimation biases are smaller than 5% for all distributions and sample dimensions.
. If the *true marginals show positive skewness*, then using symmetric marginals causes the Clayton parameter α_c to be positively biased, where the larger bias is again caused by the normal distribution. Interestingly, increasing the sample dimension to $T = 2000$ increases the estimation bias, which can reach 35% of the true value for the normal distribution. If we increase the degrees of freedom of the true DGPs to 10, the biases still remain higher than 5% when $T = 500$, and 10% when $T = 2000$, for all the symmetric distributions.

3.3. Effects of marginal and copula misspecifications on multivariate VaR estimation

Due to the large growth of trading activity and the well known trading loss of financial institutions, the regulators and supervisory committees of banks have developed and supported quantitative techniques in order to evaluate the possible losses that these institutions can incur. The most well known risk measure is the Value at Risk (VaR), which is defined as the maximum loss which can be incurred by a portfolio, at a given time horizon and at a given confidence level $(1 - p)$. While we remark that Value at Risk as a risk measure is criticized for not being sub-additive and hence may fail to stimulate diversification (see Embrechts (2000) for an overview of the criticism), still, the final Basel Capital Accord that came into force in 2007 focuses on VaR only. Therefore, we explore here the consequences of marginal and copula misspecifications on VaR estimation, by using the same DGPs as were discussed at the beginning of Section 3. For the sake of simplicity, we suppose investment of an amount $M_i = 1, i = 1, \dots, n = 3$, in every asset, and we consider an equally weighted portfolio. We make this choice for the sake of interest, since it represents the most common case in the financial literature (see, e.g., Junker and May (2005)), and because DeMiguel et al. (2007) have recently shown that a wide range of models are not consistently better than the $1/n$ rule in terms of Sharpe ratio, certainty-equivalent return, or turnover, indicating that, out of the sample, the gain from optimal diversification is more than offset by the estimation error.

We consider eight different quantiles to better highlight the overall effects of the misspecifications on the joint distribution of the losses: 0.25%, 0.50%, 1.00%, 5.00%, 95.00%, 99.00%, 99.50%, 99.75%, that is we consider both the "loss tail" and the "win tail". The full set of results for this part are reported in Fantazzini (2007), Tables 15–32. In general, the estimated quantiles show a very large degree of underestimation/overestimation, which depends heavily on both the sample dimension and the underlying joint distribution. The major findings are reported below.

- **True dependence function: Normal copula**.
  . *True marginals: Skewed marginals*
  Fat tails: $\nu = 3$. If we consider the mean VaR across Monte Carlo samples, all empirical distributions used in the analysis show overestimated VaRs, ranging from 5%–100% of the true value for the Generalized t/normal copula, to 150%–2800% for the models which use the normal distribution for the marginals, whose RMSE are usually three times the RMSE of the models which use the Generalized t and two times that of the models which employ the Student's t. Instead, if we consider the median VaR, the use of symmetric distributions now involves negative biases in the skewed tail and positive ones the other way round, ranging between 20% and 30% for the symmetric t and between 40% and 60% for the normal distribution (central quantiles 5%–95% excluded). Instead, the Generalized t marginal distribution is conservative for the skewed tail and aggressive for the other tail, with biases ranging between 5% and 20%.

  These results can be explained by looking at Tables 3–8 in Fantazzini (2007), which shows that the marginals' parameters of the conditional variance parameters ω and α are overestimated and β is underestimated for all marginal distributions, and the biases increase when moving from the Generalized t to the normal distribution. Moreover, as previously discussed in Section 3.1, when $T = 500$, there is a large difference between the estimated mean and median marginal parameters due to the poor estimates of GARCH models for within small samples. It is interesting to note that these biases more than offset the negative biases in the t copula or normal copula estimated correlations which would decrease the estimated VaR, and which do not show any substantial difference between mean and median estimates, instead, as reported in Tables 9–14 in Fantazzini (2007).

  When $T = 2000$, the biases and especially the RMSE are much smaller compared to the case for $T = 500$ for all models considered, (1)–(9). The improvement is more evident for extreme quantiles than for central quantiles. The higher precision is due to less noisy marginal estimates, since the biases in the correlations remain almost unchanged, as one can see by looking at Tables 9–14 in Fantazzini (2007). Even though the Generalized

t/normal copula is the most efficient model (as expected), the differences among copulas are not relevant when the correct marginal distribution are chosen. In particular, if the extreme quantiles in left tail are of concern, the VaR estimates delivered by the Clayton copula are very close to those of the normal copula.

Thin tails: $\nu = 10$. The biases and the RMSE are much smaller than in the case with fat tails. This result is due to the much smaller bias (and RMSE as well) in the conditional variance parameter α, which now ranges over 10%–18% instead of 20%–50% for the case with fatter tails (while the biases in the other parameters are almost unchanged). Besides, there is no major differences between the mean and median VaR estimates.

When $T = 2000$, the biases and the RMSE for correct joint distributions decrease (but the sign remains unchanged), while for misspecified models this is true for the RMSE only: the biases can actually increase. All the previous qualitative comments regarding the type of copula remain valid, even though the differences are now smaller.

. *True marginals: Symmetric marginals*

Fat tails: $\nu = 3$. As expected, the results for the Generalized t and the symmetric t are very similar, but the latter shows smaller RMSE since it is the most efficient model. As for normal marginals, the biases range over $-10\%/-40\%$ (central quantiles excluded) and they are stable across different samples.

Thin tails: $\nu = 10$. Similarly to the case with skewed marginals, the biases and the RMSE are much smaller than in the case with fat tails, due to the much smaller bias in the conditional variance parameter α, which now ranges over 12%–14% instead of 25%–40% as for the case with fatter tails.

- **True dependence function: *t* copula**.

. *True marginals: Skewed marginals*

Fat tails: $\nu = 3$. If we consider the mean VaR across Monte Carlo samples together with $T = 500$, all empirical distributions used in the analysis show overestimated VaRs. They range from 7%–80% of the Generalized t/t copula to 70%–4300% for the models which use the normal for the marginal distributions, whose RMSE are usually three/four times the RMSE of the models which use the Generalized t and two times the RMSE of the models which employ the Student's t. Instead, if we consider the median VaR, the use of symmetric distributions again involves negative biases in the skewed tail and positive ones vice versa, ranging between 20% and 30% for the symmetric t and between 40% and 60% for the normal distribution (central quantiles 5%–95% excluded). Furthermore, the Generalized t marginal distribution is conservative for the skewed tail, 0%–10%, and aggressive for the other tail, with negative biases ranging between 5% and 25%. Similarly to the case with the normal copula, these results follow from the overestimation of the conditional variance parameters ω and α and the underestimation of β, and their biases increase when moving from the Generalized t to the normal distribution. As expected, for symmetric distributions, the biases are smaller when considering the skewed tail, and larger vice versa. Furthermore, the more extreme quantiles are much better estimated than the central quantiles.

For when $T = 2000$, two results are worth noting: model (9), that is the Generalized t/Clayton copula, does much better than the Generalized t/normal copula or the Generalized t/t copula when extreme quantiles are of concern, thus confirming similar evidence with $T = 500$. However, differently from the case for $T = 500$, the mean and the median VaR estimates are much more closer and they clearly highlight that the use of symmetric distributions involves negative biases in the skewed tail and positive ones vice versa. In particular, we observe that the median VaR biases remain the same as with $T = 500$ even though the marginal parameters are now much more precisely estimated: This result is due to the severe underestimation of the t copula correlation parameters (−70% with normal marginals) and overestimation of the degrees of freedom parameter (+300% with normal marginals) when symmetric marginals are used, as reported in Table 11 in Fantazzini (2007).

Thin tails: $\nu = 10$. The biases and the RMSE are much smaller than in the case with fat tails. This result is due to the much smaller bias (and RMSE as well) in the conditional variance parameter α, which now ranges over 7%–18% instead of 20%–65% as for the case with fatter tails. Furthermore, the mean and median VaR estimates are now very close. Moreover, the use of the normal copula when the true one is the t copula delivers negative biases for almost all quantiles. The results that emerged with $T = 500$ hold also with $T = 2000$ with no major qualitative differences.

. *True marginals: Symmetric marginals*

Fat tails: $\nu = 3$. As expected, the results for the Generalized t/t copula and the symmetric t/t copula are very similar, but the latter shows smaller RMSE since it is the most efficient model. This is not true when the Clayton or the normal copula is used: in this case, the use of the Generalized t marginal delivers more precise estimates of the quantiles than the symmetric t or the normal distributions. Besides, if we look at the median VaR, the use of the normal copula delivers underestimated VaR for almost all quantiles.

Thin tails: $\nu = 10$. Similarly to the case with skewed marginals, the biases and the RMSE are much smaller than in the case with fat tails, due to the much smaller bias in the conditional variance parameter α, which now ranges over 6%–15% instead of 23%–43% as for the case with fatter tails. Again, the use of the normal copula involves underestimated VaR.

When $T = 2000$, the previous comments about the negative biases delivered by the normal copula and the normal distributions for extreme quantiles remain valid here. Furthermore, we observe that the Student's t/Clayton copula model delivers the most precise estimates when $\nu = 3$, while the Student's t/t copula model delivers the most precise estimates when $\nu = 10$.

- **True dependence function: Clayton copula**.

. *True marginals: Skewed marginals*

Fat tails: $\nu = 3$. If we consider the mean VaR across the Monte Carlo samples together with $T = 500$, almost all the empirical distributions used in the analysis show overestimated VaRs. They range from 3%–100% for the Generalized t/Clayton copula, up to 30%–600% for the models which use the normal for the marginal distributions, whose RMSE are usually three times the RMSE of the models which use the Generalized t and two times the RMSE of the models which employ the Student's t. A notable exception to this pattern is represented by the models which include the Generalized t as marginal distribution (i.e. models (3), (6) and (9)): when the data are positively skewed and the left tail is of concern, these models show a negative bias ranging between −5% and −20%. Instead, if we consider the median VaR, the use of symmetric distributions involves negative biases in the skewed tail and positive ones the other way round, similarly to what we observed when the true dependence function was represented by the normal or the t copula. Furthermore, the Generalized t marginal distribution is conservative for the skewed tail, and aggressive for the other tail. The previous comments regarding the importance of the conditional variance parameters ω and α and the minor role played by the type of copula remain unchanged. However, a major difference that we can observe when the true dependence function is modelled by a Clayton copula is the much smaller biases, RMSE and RRMSE for all models considered, (1)–(9), the normal distribution included. This result probably follows from the simpler dependence structure implied by this copula compared to the heavily parameterized normal and t copula.

When $T = 2000$, the Generalized t/t copula and Generalized t/Clayton copula models show the most precise estimates when the data are negatively skewed and left quantiles are of concern. The Generalized t/normal copula or the Generalized t/t copula is a better choice if the data are positively skewed and the right quantiles are of concern. Furthermore, we observe again that if we use symmetric marginals, we get more aggressive VaR estimates for the left quantiles if the data are negatively skewed, whereas we get more conservative estimates if the data are positively skewed. This result follows from the different bias in the Clayton copula parameter when misspecified marginals are used: a negative bias around 20% in the former case, a positive bias around 30% in the latter case; see Table 13 in Fantazzini (2007).

Thin tails: $\nu = 10$. The biases and the RMSE are much smaller than in the case with fat tails. This result is due to the much smaller bias (and RMSE as well) in the conditional variance parameter α, which now ranges over 8%–19% instead of 20%–55% as for the case with fatter tails. Again, the biases in the correlations and, in general, the choice of the copula play a residual role when $T = 500$.

Finally, when $T = 2000$, the Generalized t/Clayton copula and the Generalized t/t copula are the models with lowest RMSE, but the latter show biases around zero, while the former show positive biases between 5% and 10%, that is the VaR estimates are more conservative.

. *True marginals: Symmetric marginals*

Fat tails: $\nu = 3$. As expected, the results for the Generalized t/Clayton copula and the symmetric t/Clayton copula are very similar, but the latter shows smaller RMSE since it is the most efficient model. This is not true when the t copula or the normal copula (for extreme quantiles) is used: the use of the Generalized t marginal delivers more precise estimates of the quantiles.

Thin tails: $\nu = 10$. Similarly to the case with skewed marginals, the biases and the RMSE are much smaller than in the case with fat tails, due to the smaller bias in the conditional variance parameter α, which now ranges over 11%–15% instead of 19%–27% as for the case with fatter tails. The use of the true Clayton copula results in VaR estimates that are almost unbiased (biases smaller than 1%), while the biases for the other two copulas range between 10% and 20%. This is in sharp contrast with what we saw when the true dependence was given by the t or normal copula, where the differences among copulas were minor. Instead, similarly to the previous cases, the mean and median VaR estimates are now very close.

When $T = 2000$, as expected, the Student's t/Clayton model shows the lowest RMSE and its biases are around zero, differently from the Student's t/normal copula and Student's t/t copula which show negative biases between 5% and 10%, instead. Moreover, the mean and median VaR estimates are close when $\nu = 3$, too, similarly to what we observed with the normal and the t copula.

3.4. Discussion

In order to summarize the previous findings, we report in Table 2 the Min and the Max biases in the mean VaR estimates for all the nine multivariate specifications, across all quantiles. We consider the case for $T = 2000$ for the sake of interest, since this sample dimension will be used in the following empirical analysis. Moreover, we have just shown that the estimated mean and median VaR are very close when using large samples.

The simulation results presented in the previous section indicate that, when small samples are considered and the data are leptokurtic, the overestimation/underestimation in the marginal GARCH parameters is so large as to deliver conservative VaR estimates even with a simple multivariate normal distribution (model (1)). In general, the VaR estimates are very poor and suffer from computational problems when estimating GARCH models for small samples, as also discussed in Hwang and Valls Pereira (2006). When the sample dimension increases, the biases in the volatility parameters are much smaller, whereas those in the copula parameters remain almost unchanged or even increase: see, for example, the t copula and the Clayton copula parameters, when symmetric marginals are used instead of skewed ones. In this case, copula misspecifications do play a role. However, these effects depend heavily on the sign of the dependence: if it is negative, the bias can be as large as 70%, like for t copula correlations, for example. If it is positive, the bias is much smaller (10% or less for the t copula correlations), and the effects on quantile estimation are much more limited, if not completely offset by marginal misspecifications. Therefore, this Monte Carlo evidence gives some insights into why previous empirical literature found that the influence of a misspecification in the copula is given with 20% or less of the whole estimation error for the VaR; see e.g. Ané and Kharoubi (2003) and Junker and May (2005).

4. Empirical analysis

In order to compare the nine multivariate distribution specifications proposed in Table 1 and analyzed in detail in Section 3, we measured the Value at Risk of ten trivariate portfolios. These portfolios are composed of assets from the Dow Jones Industrial Index, with daily data taking into consideration the very volatile period between March 1996 and November 2003. Following Giacomini and Komunjer (2005) and González-Rivera et al. (2006), we use a rolling forecasting scheme of 2000 observations, because it may be more robust against a possible parameter variation. Furthermore, we chose this time dimension because the previous simulation studies in Section 3 clearly highlighted that the biases and the RMSE of the VaR estimates are much smaller and they are close to zero for correctly specified joint distributions. In our case we have 3000 observations so we split the sample in this way: 2000 observations for the estimation window and 1000 for the out-of-sample evaluation. Besides, Christoffersen and Diebold (2000) and Giot and Laurent (2003) showed that volatility forecastability decays quickly with the time horizon of the forecasts. An immediate consequence is that volatility forecastability is relevant for short time horizons (such as daily trading) but not for long time horizons. Therefore, we focused on daily returns and VaR performances for daily trading portfolios, only. A general algorithm for estimating the 0.25%, 0.5%, 1%, 5%, 95%, 99%, 99.5% and 99.75% VaR over a one-day holding period for a portfolio P of n assets with invested positions equal to M_i, $i = 1, \ldots, n$, is the following:

(1) Given a set of estimated parameter values at time $t-1$, simulate 100,000 scenarios for each asset log-return, $\{y_{1,t}, \ldots, y_{n,t}\}$, over the time horizon $[t-1, t]$, using a general multivariate distribution as in Table 1, by using this procedure:
   (a) Firstly, *generate n random variates* $(u_{1,t}, \ldots, u_{n,t})$ *from the copula* $\hat{C}_t$ forecast at time t, which can be normal, Student t, or Clayton.
   (b) Secondly, get a vector $n \times 1\mathbf{Q_t}$ of *standardized asset log-returns* $\eta_{i,t}$ by using the inverse functions of the forecast marginals at time t, which can be symmetric or skewed:
$$\mathbf{Q_t} = \left(\eta_{1,t}, \ldots, \eta_{n,t}\right) = \left(F_1^{-1}(u_{1,t}; \hat{\delta}_1), \ldots, F_n^{-1}(u_{n,t}; \hat{\delta}_n)\right).$$

Table 2
VaR Bias: Min and Max in % when $T = 2000$ across all quantiles

	True copula: Normal copula				True copula: *t* copula				True copula: Clayton			
	Skewed m.		Sym. m.		Skewed m.		Sym. m.		Skewed m.		Sym. m.	
	$\nu = 3$	$\nu = 10$	$\nu = 3$	$\nu = 10$	$\nu = 3$	$\nu = 10$	$\nu = 3$	$\nu = 10$	$\nu = 3$	$\nu = 10$	$\nu = 3$	$\nu = 10$
Model (1)	−10/891	−25/31	−9/208	−10/3	12/1454	−39/48	−20/263	−30/10	−25/231	−32/31	−30/77	−23/12
Model (2)	−111/333	−54/76	2/12	0/1	−106/583	−52/95	−4/35	−21/7	−50/103	−32/55	−10/31	−11/23
Model (3)	−39/24	−25/13	6/35	0/1	−47/42	−41/14	−5/33	−21/8	−45/46	−35/36	−9/37	−11/24
Model (4)	−1/992	−24/30	−7/217	−9/2	−42/1496	−22/77	−10/289	−16/5	−32/184	−30/33	−41/32	−23/12
Model (5)	−115/384	−55/75	4/23	0/2	−112/528	−50/98	3/18	−2/0	−52/106	−31/60	−7/38	−11/26
Model (6)	−39/36	−24/14	1/6	0/2	−56/35	−25/14	2/15	−1/0	−44/50	−34/39	−9/32	−11/26
Model (7)	3/1130	−19/46	−9/208	−5/12	0/1918	−28/77	1/375	−16/34	−27/243	−28/23	−30/72	−15/3
Model (8)	−99/438	−46/97	0/26	−1/11	−98/739	−36/125	−3/75	−11/31	−47/100	−28/45	1/3	0/1
Model (9)	−29/34	−17/16	4/33	−1/12	−35/59	−29/32	1/86	−11/31	−36/24	−28/11	1/4	0/0

(c) Thirdly, *rescale the standardized asset log-returns* by using the forecast means and variances, estimated with AR–GARCH models:

$$\{y_{1,t}, \ldots, y_{n,t}\} = \left(\hat{\mu}_{1,t} + \eta_{1,t} \cdot \sqrt{\hat{h}_{1,t}}, \quad \ldots, \quad \hat{\mu}_{n_t} + \eta_{n,t} \cdot \sqrt{\hat{h}_{n,t}}\right). \tag{6}$$

(d) Finally, repeat this procedure $j = 1, \ldots, 100{,}000$ times.

(2) By using these 100,000 scenarios, the portfolio P is re-evaluated at time t; that is,

$$P_t^j = M_{1,t-1} \cdot \exp(y_{1,t}) + \ldots + M_{n,t-1} \cdot \exp(y_{n,t}), \quad j = 1, \ldots, 100{,}000. \tag{7}$$

(3) Portfolio *losses* in each scenario j are then computed (possible profits are considered as negative losses):

$$\text{Loss}_j = P_{t-1} - P_t^j, \quad j = 1, \ldots, 100{,}000. \tag{8}$$

(4) The calculation of the 0.25%, 0.5%, 1%, 5%, 95%, 99%, 99.5% and 99.75% VaR is now straightforward:
   (a) Order the 100,000 Loss_j in increasing order.
   (b) The pth VaR is the $(1-p)\cdot 100{,}000$th ordered scenario, where $p = \{0.25\%, 0.5\%, 1\%, 5\%, 95\%, 99\%, 99.5\%, 99.75\%\}$. For example, the 0.25% VaR is the 99,750th ordered scenario.

The predicted one-step-ahead VaR forecasts were then compared with the observed portfolio losses and both results were recorded for later assessment.

4.1. VaR evaluation

We compare the different multivariate models by looking at their VaR forecasts, using Hansen's (2005) Superior Predictive Ability (SPA) test together with the Giacomini and Komunjer (2005) asymmetric loss function

$$\mathcal{T}_p(e_{t+1}) \equiv (p - \mathbb{1}(e_{t+1} < 0))e_{t+1}, \tag{9}$$

where $e_{t+1} = L_{t+1} - \widehat{\text{VaR}_{t+1|t}}$, $\mathbb{1}$ is the indicator function, L_{t+1} is the realized loss, while $\widehat{\text{VaR}_{t+1|t}}$ is the VaR forecast at time $t+1$ on information available at time t. We also employ Kupiec's (1995) unconditional coverage test and Christoffersen's (1998) conditional coverage test, given their importance in the empirical literature, even though their power can be very low.

4.2. VaR out-of-sample results

The actual VaR exceedances N/T, the p-values p_{UC} of Kupiec's Unconditional Coverage test, and the p-values p_{CC} of Christoffersen's Conditional Coverage test, for the VaR forecasts at all probability levels are reported in Tables 3 and 4. Instead, Tables 5 and 6 report the asymmetric loss functions (9) and the p-values of the SPA test, for all the quantiles and for long and short positions. The previous tables show that the ten portfolios present a wide range of joint distributions. The major insights are the following ones:

- 1st *portfolio: 3M, AT&T, Alcoa*: The symmetric Student's t/Clayton copula model represents the most appropriate choice according to all tests and the asymmetric loss distributions. However, the Generalized t/Clayton copula model cannot be outperformed for all quantiles, too, according to Hansen's tests. The simulation results reported in Table 32 in Fantazzini (2007) show that the use of symmetric marginals together with the true Clayton copula, when the true margins are positively skewed and the marginal degrees of freedom are not too low, results in positive biases for the left quantiles and negative ones for the right quantiles. Instead, the same table shows that the true Generalized t marginals produce smaller biases but with the opposite signs. Therefore, this MC evidence explains why the empirical ratio of VaR exceedances N/T is smaller for the Student's t compared to the Generalized t when dealing with long positions, while the reverse is true for short positions. Besides, the empirical exception frequencies N/T in Table 3 show that the symmetric Student's T/t copula and the symmetric Student's t/normal copula deliver aggressive VaR estimates when dealing with long positions, whereas conservative if short positions are of concern. This evidence can be explained by looking at Tables 20 and 25 in Fantazzini (2007), where simulation results show that when the true dependence structure is represented by the Clayton copula but elliptical copulas are used, the resulting VaR are underestimated for long positions and overestimated for short positions.

- 2nd *portfolio: Altria Group, American Express, Boeing*. The Generalized t/normal copula model turns out to be the best joint distribution, but the Student's t/normal copula performs rather well, too. It is interesting to note that the models using the t copula and the Clayton copula produce conservative VaR estimates for the extreme left quantiles: this is line with the simulation evidence reported in Tables 24 and 30 in Fantazzini (2007), where the use of these copulas when the true one is the normal copula results in VaR estimates which are 5%–10% higher than the true ones, given that the true marginals are symmetric with low degrees of freedom.
- 3rd *portfolio: Caterpillar, Citigroup, Coca Cola*. Like for the previous portfolio, the Student's t/normal copula model is already a proper choice, but the models with the t copula and Clayton copula follow closely. The same comments about left quantiles discussed for the 2nd portfolio apply here, too.
- 4th *portfolio: DuPont, Eastman Kodak, Exxon Mobil*. The Generalized t/t copula, i.e. model (6), seems to be the preferred model according to loss functions and Hansen tests. However, the Kupiec and Christoffersen tests highlight that it is slightly too aggressive when central left quantiles are of concern: such evidence can be explained by looking at Table 25 in Fantazzini (2007), which shows that when there is positive marginal skewness and the true dependence structure is the t copula, model (6) shows negative biases for left quantiles, particularly for central quantiles, and positive ones for right quantiles (even though the RRMSE are very small). For the same D.G.P., the previous table shows that a symmetric Student's t/t copula model delivers higher biases but with opposite signs, instead: this is exactly what we observe in Table 3, where model (5) performs slightly better for left quantiles, but much worse for right quantiles.
- 5th *portfolio: General Electric, General Motors, Hewlett-Packard*. The Student's t/t copula is the model that is able to pass all tests and it shows the lowest loss functions for many quantiles, too. Instead, the Student's t/Clayton performs better if left quantiles are of concern: the Monte Carlo results reported in Table 31 in Fantazzini (2007) show that if the Clayton copula is used instead of the true t copula (and the true marginals show no asymmetry), the former copula delivers overestimated VaR estimates for the left quantiles, whereas it delivers underestimated ones for the right quantiles.
- 6th *portfolio: Home Depot, Honeywell, Intel*. The Student's t/Clayton copula is the best model for long positions, while the Student's t/t copula is preferred for short positions, instead. However, both models cannot be outperformed for all quantiles, according to the Hansen's test. Furthermore, the Generalized t/Clayton model delivers close results, too. This case is very similar to that of the first portfolio and the same comments apply.
- 7th *portfolio: IBM, Intl. Paper, JPMorgan Chase & Co*. Again, the Student's t/Clayton copula is the best model for long positions, while the Student's t/t copula is preferred for short positions, instead. This case is very similar to those of the first and the sixth portfolios and the same comments apply.
- 8th *portfolio: Johnson & Johnson, McDonald's, Merck*. The simple multivariate normal model is already a proper choice, as it passes all tests and its loss functions are very low, if not the lowest. Therefore, it does not come as a surprise that the models using the t copula and the Clayton copula tend to be too conservative, particularly for the left quantiles, thus confirming the MC evidence reported in tables 24 and 30 in Fantazzini (2007).
- 9th *portfolio: Microsoft, Procter & Gamble, SBC*. The Student's t/normal copula model results to be the best choice. This portfolio presents a joint distribution which is very similar to the one estimated for the previous second and third portfolios and the same comments apply.
- 10th *portfolio: United Technologies, Wal-Mart, Walt Disney*. The symmetric Student's t/t copula, i.e. model (5), is the preferred model according to loss functions and statistical tests, whereas the Student's t/Clayton copula model is more conservative for the 0.25% left quantile: this portfolio is therefore similar to the fifth portfolio and the same comments apply.

The previous results indicate that a Generalized t or a symmetric t distribution with a GARCH(1, 1) specification for the variance, together with a Clayton copula, can be a good choice for precise VaR estimates for long positions. A symmetric t/normal copula model is a better choice if short positions are of concern, instead. Finally, a Generalized t/t copula model can be a good compromise if both long and short positions are of interest.

5. Conclusions

This paper investigated how misspecification both in the marginals and in the copulas may affect the estimation of the Value at Risk when dealing with multivariate portfolios. We first performed a Monte Carlo study to assess the potential impact of misspecified margins on the estimation of the copula parameters under different hypotheses for

Table 3
VaR exceedances N/T, Kupiec's and Christoffersen's tests: 1st–5th portfolios

	N/T 0.25%	puc	pcc	N/T 0.50%	puc	pcc	N/T 1%	puc	pcc	N/T 5%	puc	pcc	N/T 95%	puc	pcc	N/T 99%	puc	pcc	N/T 99.5%	puc	pcc	N/T 99.75%	puc	pcc
P1M1	1.60%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	3.40%	**0.00**	**0.00**	8.30%	**0.00**	**0.00**	1.10%	**0.00**	**0.00**	1.60%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	6.50%	**0.04**	0.08
P1M2	0.40%	0.38	0.67	0.80%	0.22	0.44	1.70%	**0.04**	0.07	6.60%	**0.03**	0.08	0.20%	0.74	0.94	0.50%	1.00	0.07	1.00%	**0.03**	0.90	5.60%	0.39	0.61
P1M3	0.50%	0.16	0.37	1.20%	**0.01**	**0.03**	2.30%	**0.00**	**0.00**	7.40%	**0.00**	**0.00**	0.10%	0.28	0.56	0.40%	0.64	0.07	0.90%	0.75	0.87	4.80%	0.77	0.54
P1M4	1.50%	**0.00**	**0.00**	2.10%	**0.00**	**0.00**	3.30%	**0.00**	**0.00**	8.30%	**0.00**	**0.00**	1.00%	**0.00**	**0.00**	1.50%	**0.00**	0.08	2.10%	**0.00**	**0.00**	6.80%	**0.01**	**0.04**
P1M5	0.40%	0.38	0.67	0.70%	0.40	0.67	1.60%	0.08	0.17	6.70%	**0.02**	0.06	0.10%	0.28	0.56	0.50%	1.00	0.07	1.20%	0.54	0.71	5.60%	0.39	0.61
P1M6	0.40%	0.38	0.67	1.10%	**0.02**	0.06	1.90%	**0.01**	**0.03**	8.00%	**0.00**	**0.00**	0.10%	0.28	0.56	0.20%	0.13	0.08	0.80%	0.51	0.75	5.00%	1.00	0.65
P1M7	1.20%	**0.00**	**0.00**	1.80%	**0.00**	**0.00**	3.30%	0.00	0.00	8.50%	0.00	0.00	1.70%	**0.00**	**0.00**	2.30%	**0.00**	0.09	3.30%	**0.00**	**0.00**	9.00%	**0.00**	**0.00**
P1M8	0.20%	0.74	0.94	0.40%	0.64	0.88	1.10%	0.75	0.84	5.70%	0.32	0.60	0.50%	0.16	0.37	0.70%	0.40	0.06	2.00%	**0.01**	**0.01**	6.30%	0.07	0.03
P1M9	0.20%	0.74	0.94	0.50%	1.00	0.98	1.30%	0.36	0.56	6.80%	**0.01**	0.05	0.40%	0.38	0.67	0.50%	1.00	0.07	1.40%	0.23	0.40	5.90%	0.20	0.19
P2M1	0.70%	**0.02**	0.06	1.00%	0.05	0.13	1.50%	0.14	0.27	6.90%	**0.01**	**0.03**	1.10%	**0.00**	**0.00**	1.20%	**0.01**	**0.03**	1.80%	**0.02**	0.05	5.70%	0.32	0.56
P2M2	0.10%	0.28	0.56	0.20%	0.13	0.31	0.90%	0.75	0.87	6.70%	**0.02**	0.05	0.50%	0.16	0.37	0.50%	1.00	0.07	1.30%	**0.02**	0.24	5.60%	0.39	0.41
P2M3	0.10%	0.28	0.56	0.30%	0.33	0.62	0.90%	0.75	0.87	6.60%	**0.03**	0.07	0.40%	0.38	0.67	0.50%	1.00	0.07	1.10%	0.75	0.84	5.30%	0.67	0.41
P2M4	0.60%	0.06	0.17	1.00%	0.05	0.13	1.60%	0.08	0.17	7.20%	**0.00**	**0.01**	0.90%	**0.00**	**0.01**	1.20%	**0.01**	0.07	1.70%	**0.04**	0.07	5.70%	0.32	0.56
P2M5	0.10%	0.28	0.56	0.10%	**0.03**	0.09	0.90%	0.75	0.87	6.70%	**0.02**	0.05	0.40%	0.38	0.67	0.50%	1.00	0.07	1.20%	0.54	0.71	5.80%	0.26	0.35
P2M6	0.10%	0.28	0.56	0.20%	0.13	0.31	0.90%	0.75	0.87	6.70%	**0.02**	0.05	0.30%	0.76	0.95	0.50%	1.00	0.07	0.80%	0.51	0.75	5.30%	0.67	0.41
P2M7	0.60%	0.06	0.17	1.10%	**0.02**	0.06	1.80%	0.02	0.05	7.90%	**0.00**	**0.00**	1.50%	**0.00**	**0.00**	2.00%	**0.00**	0.08	2.60%	**0.00**	**0.00**	7.30%	**0.00**	**0.01**
P2M8	0.10%	0.28	0.56	0.10%	**0.03**	0.09	0.60%	0.17	0.38	6.40%	0.05	0.12	0.50%	0.16	0.37	1.00%	0.05	0.06	1.50%	0.14	0.16	6.40%	0.05	0.13
P2M9	0.10%	0.28	0.56	0.10%	**0.03**	0.09	0.80%	0.51	0.75	6.60%	**0.03**	0.07	0.50%	0.16	0.37	0.80%	0.22	0.07	1.60%	0.08	0.11	6.10%	0.12	0.24
P3M1	0.60%	0.06	**0.01**	1.30%	**0.00**	**0.00**	2.10%	**0.00**	**0.01**	5.80%	0.26	0.15	0.70%	**0.02**	**0.01**	1.00%	0.05	**0.03**	1.40%	0.23	0.20	5.40%	0.57	0.20
P3M2	0.40%	0.38	**0.01**	0.50%	1.00	0.05	1.40%	0.23	0.20	4.90%	0.88	0.56	0.10%	0.28	0.56	0.30%	0.33	0.05	1.00%	0.88	0.23	4.80%	0.77	0.86
P3M3	0.40%	0.38	**0.01**	0.60%	0.66	0.07	1.40%	0.23	0.20	5.40%	0.57	0.71	0.10%	0.28	0.56	0.40%	0.64	0.05	0.90%	0.75	0.17	4.40%	0.37	0.51
P3M4	0.50%	0.16	**0.01**	1.40%	0.00	**0.00**	2.00%	**0.01**	**0.01**	5.90%	0.20	0.12	0.60%	0.06	**0.01**	1.00%	0.05	0.06	1.40%	0.23	0.20	5.40%	0.57	0.20
P3M5	0.30%	0.76	**0.01**	0.40%	0.64	**0.03**	1.10%	0.75	0.26	5.20%	0.77	0.86	0.10%	0.28	0.56	0.30%	0.33	0.05	1.00%	1.00	0.23	4.80%	0.77	0.86
P3M6	0.40%	0.38	**0.01**	0.50%	1.00	0.05	1.20%	0.54	0.26	5.70%	0.32	0.45	0.10%	0.28	0.56	0.20%	0.13	0.06	0.70%	0.31	0.57	4.40%	0.37	0.51
P3M7	0.50%	0.16	**0.01**	0.90%	0.11	0.05	1.90%	0.01	**0.03**	6.00%	0.16	0.23	1.40%	**0.00**	**0.00**	2.10%	**0.00**	0.06	2.90%	**0.00**	**0.00**	7.20%	**0.00**	**0.01**
P3M8	0.10%	0.28	0.56	0.40%	0.64	**0.03**	0.60%	0.17	**0.03**	4.60%	0.56	0.57	0.60%	0.06	0.17	1.00%	0.05	0.05	2.00%	**0.01**	**0.01**	6.10%	0.12	0.24
P3M9	0.20%	0.74	**0.01**	0.40%	0.64	**0.03**	0.80%	0.51	0.12	5.30%	0.67	0.79	0.40%	0.38	0.67	0.90%	0.11	0.05	1.50%	0.14	0.16	5.70%	0.32	0.38
P4M1	1.00%	**0.00**	**0.00**	1.90%	**0.00**	**0.00**	2.50%	**0.00**	**0.00**	6.10%	0.12	0.18	1.00%	**0.00**	**0.00**	1.20%	**0.01**	**0.03**	1.70%	**0.04**	0.10	6.20%	0.09	0.07
P4M2	0.40%	0.38	0.67	0.70%	0.40	0.67	2.10%	0.00	0.01	6.60%	**0.03**	0.08	0.80%	**0.01**	**0.02**	1.10%	**0.02**	0.07	1.20%	**0.03**	0.71	6.30%	0.07	0.11
P4M3	0.60%	0.06	0.17	0.90%	0.11	0.25	2.10%	0.00	0.01	6.90%	**0.01**	0.03	0.60%	0.06	0.17	1.00%	0.05	0.07	1.20%	0.54	0.71	6.20%	0.09	0.13
P4M4	1.10%	**0.00**	**0.00**	2.00%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	6.20%	0.09	0.13	1.00%	**0.00**	**0.00**	1.10%	0.02	0.06	1.80%	**0.02**	0.05	6.50%	**0.04**	**0.02**
P4M5	0.20%	0.74	0.94	0.70%	0.40	0.67	2.00%	**0.01**	**0.01**	6.80%	**0.01**	0.04	0.70%	**0.02**	0.06	1.10%	0.02	0.07	1.20%	0.54	0.71	6.80%	**0.01**	**0.02**
P4M6	0.40%	0.38	0.67	0.80%	0.22	0.44	2.10%	**0.00**	**0.01**	7.00%	**0.01**	0.02	0.50%	0.16	0.37	0.90%	0.11	0.07	1.10%	0.75	0.84	6.20%	0.09	0.13
P4M7	1.20%	**0.00**	**0.00**	2.00%	**0.00**	**0.00**	2.90%	**0.00**	**0.00**	7.60%	**0.00**	**0.00**	1.30%	**0.00**	**0.00**	2.40%	**0.00**	0.08	3.10%	**0.00**	**0.00**	8.10%	**0.00**	**0.00**

(*continued on next page*)

Table 3 (*continued*)

	N/T 0.25%	p_{UC}	p_{CC}	N/T 0.50%	p_{UC}	p_{CC}	N/T 1%	p_{UC}	p_{CC}	N/T 5%	p_{UC}	p_{CC}	N/T 95%	p_{UC}	p_{CC}	N/T 99%	p_{UC}	p_{CC}	N/T 99.5%	p_{UC}	p_{CC}	N/T 99.75%	p_{UC}	p_{CC}
P4M8	0.20%	0.74	0.94	0.50%	1.00	0.98	1.50%	0.14	0.27	6.40%	0.05	0.13	0.90%	**0.00**	**0.01**	1.10%	**0.02**	0.06	2.00%	**0.01**	**0.01**	7.90%	**0.00**	**0.00**
P4M9	0.20%	0.74	0.94	0.60%	0.66	0.88	1.90%	0.01	0.03	6.80%	**0.01**	**0.04**	0.90%	**0.00**	**0.01**	1.10%	**0.02**	0.07	1.70%	**0.04**	0.10	7.40%	**0.00**	**0.00**
P5M1	1.40%	**0.00**	**0.00**	2.00%	**0.00**	**0.00**	2.50%	**0.00**	**0.00**	7.80%	**0.00**	**0.00**	1.00%	**0.00**	**0.00**	1.70%	**0.00**	**0.00**	2.50%	**0.00**	**0.00**	6.70%	**0.02**	0.05
P5M2	0.70%	**0.02**	0.06	1.30%	**0.00**	**0.01**	1.70%	**0.04**	0.07	6.40%	0.05	0.07	0.30%	0.76	0.95	0.60%	0.66	0.06	1.00%	0.05	0.90	6.30%	0.07	0.17
P5M3	0.70%	**0.02**	0.06	1.30%	**0.00**	**0.01**	1.70%	**0.04**	0.07	6.70%	**0.02**	**0.02**	0.30%	0.76	0.95	0.40%	0.64	0.07	1.00%	1.00	0.90	5.90%	0.20	0.43
P5M4	1.40%	**0.00**	**0.00**	2.00%	**0.00**	**0.00**	2.60%	**0.00**	**0.00**	7.90%	**0.00**	**0.00**	0.90%	**0.00**	**0.01**	1.50%	**0.00**	0.08	2.50%	**0.00**	**0.00**	6.80%	**0.01**	**0.04**
P5M5	0.50%	0.16	0.37	1.00%	0.05	0.13	1.50%	0.14	0.27	6.40%	0.05	0.07	0.30%	0.76	0.95	0.50%	1.00	0.06	1.00%	1.00	0.90	6.30%	0.07	0.17
P5M6	0.60%	0.06	0.17	1.10%	**0.02**	0.06	1.60%	0.08	0.17	6.70%	**0.02**	**0.02**	0.30%	0.76	0.95	0.40%	0.64	0.07	0.90%	0.75	0.87	6.00%	0.16	0.36
P5M7	1.40%	**0.00**	**0.00**	1.70%	**0.00**	**0.00**	2.70%	**0.00**	**0.00**	8.50%	**0.00**	**0.00**	2.00%	**0.00**	**0.00**	3.20%	**0.00**	0.09	4.30%	**0.00**	**0.00**	9.50%	**0.00**	**0.00**
P5M8	0.30%	0.76	0.95	0.60%	0.66	0.88	1.30%	0.36	0.56	5.90%	0.20	0.29	0.60%	0.06	0.17	1.00%	0.05	0.06	2.00%	**0.01**	**0.01**	7.20%	**0.00**	**0.01**
P5M9	0.30%	0.76	0.95	0.90%	0.11	0.25	1.40%	0.23	0.40	6.40%	0.05	0.07	0.50%	0.16	0.37	1.00%	0.05	0.06	1.70%	**0.04**	0.10	7.00%	**0.01**	**0.02**

Table 4
VaR exceedances N/T, Kupiec's and Christoffersen's tests: 6th–10th portfolios

	N/T 0.25%	p_{UC}	p_{CC}	N/T 0.50%	p_{UC}	p_{CC}	N/T 1%	p_{UC}	p_{CC}	N/T 5%	p_{UC}	p_{CC}	N/T 95%	p_{UC}	p_{CC}	N/T 99%	p_{UC}	p_{CC}	N/T 99.5%	p_{UC}	p_{CC}	N/T 99.75%	p_{UC}	p_{CC}
P6M1	1.80%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	3.30%	**0.00**	**0.00**	7.90%	**0.00**	**0.00**	0.90%	**0.00**	**0.01**	1.50%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	7.30%	**0.00**	**0.00**
P6M2	0.30%	0.76	0.95	1.20%	**0.01**	**0.03**	2.40%	**0.00**	**0.00**	6.70%	**0.02**	0.05	0.50%	0.16	0.37	0.80%	0.22	0.07	1.10%	**0.02**	0.84	5.30%	0.67	0.41
P6M3	0.40%	0.38	0.67	1.20%	**0.01**	**0.03**	2.50%	**0.00**	**0.00**	7.00%	**0.01**	**0.02**	0.40%	0.38	0.67	0.60%	0.66	0.07	1.10%	0.75	0.84	5.20%	0.77	0.17
P6M4	1.70%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	3.10%	**0.00**	**0.00**	8.30%	**0.00**	**0.00**	0.90%	**0.00**	**0.01**	1.50%	**0.00**	0.08	2.10%	**0.00**	**0.01**	7.50%	**0.00**	**0.00**
P6M5	0.30%	0.76	0.95	1.00%	0.05	0.13	2.20%	**0.00**	**0.00**	6.80%	**0.01**	**0.04**	0.30%	0.76	0.95	0.60%	0.66	0.07	1.10%	0.75	0.84	5.50%	0.47	0.21
P6M6	0.30%	0.76	0.95	1.10%	**0.02**	0.06	2.50%	**0.00**	**0.00**	7.30%	**0.00**	**0.01**	0.30%	0.76	0.95	0.60%	0.66	0.07	1.00%	1.00	0.90	5.10%	0.88	0.15
P6M7	2.00%	**0.00**	**0.00**	2.60%	**0.00**	**0.00**	3.90%	**0.00**	**0.00**	9.10%	**0.00**	**0.00**	1.90%	**0.00**	**0.00**	2.90%	**0.00**	0.09	3.80%	**0.00**	**0.00**	10.30%	**0.00**	**0.00**
P6M8	0.30%	0.76	0.95	0.50%	1.00	0.98	1.80%	**0.02**	0.05	6.80%	**0.01**	0.05	0.80%	**0.01**	**0.02**	1.20%	**0.01**	0.07	2.00%	**0.01**	**0.01**	7.50%	**0.00**	**0.00**
P6M9	0.30%	0.76	0.95	0.60%	0.66	0.88	1.80%	**0.02**	0.05	6.80%	**0.01**	**0.04**	0.80%	**0.01**	**0.02**	1.00%	0.05	0.07	1.90%	**0.01**	**0.03**	7.20%	**0.00**	**0.01**
P7M1	1.90%	**0.00**	**0.00**	2.50%	**0.00**	**0.00**	3.30%	**0.00**	**0.00**	7.10%	**0.00**	**0.01**	1.00%	**0.00**	**0.00**	1.70%	**0.00**	**0.00**	2.50%	**0.00**	**0.00**	6.20%	0.09	0.24
P7M2	0.50%	0.16	0.37	0.80%	0.22	0.44	2.20%	**0.00**	**0.00**	7.10%	**0.00**	**0.01**	0.20%	0.74	0.94	0.60%	0.66	0.07	1.00%	**0.00**	0.90	6.30%	0.07	0.19
P7M3	0.50%	0.16	0.37	1.00%	0.05	0.13	2.10%	**0.00**	**0.01**	7.10%	**0.00**	**0.01**	0.20%	0.74	0.94	0.20%	0.13	0.07	1.00%	1.00	0.90	6.10%	0.12	0.30
P7M4	1.80%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	3.20%	**0.00**	**0.00**	7.50%	**0.00**	**0.00**	1.00%	**0.00**	**0.00**	1.60%	**0.00**	0.08	2.70%	**0.00**	**0.00**	6.20%	0.09	0.24
P7M5	0.40%	0.38	0.67	0.60%	0.66	0.88	2.10%	**0.00**	**0.01**	7.20%	**0.00**	**0.01**	0.20%	0.74	0.94	0.40%	0.64	0.07	1.00%	1.00	0.90	6.30%	0.07	0.19
P7M6	0.40%	0.38	0.67	0.60%	0.66	0.88	2.00%	**0.01**	**0.01**	7.20%	**0.00**	**0.01**	0.10%	0.28	0.56	0.20%	0.13	0.07	0.90%	0.75	0.87	6.20%	0.09	0.24
P7M7	1.50%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	3.20%	**0.00**	**0.00**	7.60%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	2.90%	**0.00**	0.08	3.70%	**0.00**	**0.00**	8.00%	**0.00**	**0.00**
P7M8	0.20%	0.74	0.94	0.40%	0.64	0.88	1.40%	0.23	0.40	6.50%	**0.04**	0.11	0.60%	0.06	0.17	0.90%	0.11	0.07	2.10%	**0.00**	**0.00**	6.90%	**0.01**	**0.03**
P7M9	0.20%	0.74	0.94	0.60%	0.66	0.88	1.50%	0.14	0.27	6.60%	**0.03**	0.08	0.30%	0.76	0.95	0.70%	0.40	0.07	2.10%	**0.00**	**0.00**	6.90%	**0.01**	**0.03**
P8M1	0.30%	0.76	**0.01**	0.60%	0.66	0.07	1.00%	1.00	0.23	3.70%	0.05	0.02	0.40%	0.38	0.67	1.00%	0.05	0.13	1.60%	0.08	0.11	4.80%	0.77	0.54
P8M2	0.20%	0.74	0.94	0.20%	0.13	0.31	0.90%	0.75	0.17	4.10%	0.18	0.11	0.30%	0.76	0.95	0.50%	1.00	**0.04**	1.20%	0.18	0.26	4.50%	0.46	0.33
P8M3	0.20%	0.74	0.94	0.30%	0.33	0.01	0.90%	0.75	0.17	4.30%	0.30	0.07	0.30%	0.76	0.95	0.30%	0.33	**0.04**	1.10%	0.75	0.26	4.50%	0.46	0.33
P8M4	0.20%	0.74	0.94	0.60%	0.66	0.07	1.00%	1.00	0.23	3.80%	0.07	0.03	0.40%	0.38	0.67	1.00%	0.05	**0.04**	1.50%	0.14	0.16	4.80%	0.77	0.54
P8M5	0.00%	**0.03**	0.08	0.20%	0.13	0.31	0.70%	0.31	0.07	4.10%	0.18	**0.04**	0.30%	0.76	0.95	0.30%	0.33	**0.04**	1.20%	0.54	0.26	4.50%	0.46	0.33
P8M6	0.10%	0.28	0.56	0.30%	0.33	**0.01**	0.90%	0.75	0.17	4.50%	0.46	0.01	0.30%	0.76	0.95	0.30%	0.33	0.05	1.10%	0.75	0.26	4.50%	0.46	0.33
P8M7	0.10%	0.28	0.56	0.30%	0.33	0.62	0.90%	0.75	0.17	4.00%	0.13	0.08	1.50%	**0.00**	**0.00**	1.90%	**0.00**	**0.04**	2.70%	**0.00**	**0.00**	6.20%	0.09	0.07
P8M8	0.00%	**0.03**	0.08	0.10%	**0.03**	0.09	0.30%	**0.01**	**0.00**	3.60%	**0.03**	**0.01**	0.80%	**0.01**	**0.02**	1.40%	**0.00**	**0.04**	2.00%	**0.01**	**0.00**	5.90%	0.20	0.09
P8M9	0.00%	**0.03**	0.08	0.10%	**0.03**	0.09	0.60%	0.17	0.03	3.90%	0.10	0.05	0.60%	0.06	0.17	1.30%	**0.00**	**0.04**	1.80%	**0.02**	**0.01**	5.50%	0.47	0.08
P9M1	0.90%	**0.00**	**0.01**	1.50%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	6.40%	0.05	0.15	0.90%	**0.00**	**0.01**	1.00%	0.05	0.13	1.10%	0.75	0.84	4.40%	0.37	0.51
P9M2	0.50%	0.16	0.37	1.00%	0.05	0.13	1.60%	0.08	0.17	6.40%	0.05	0.12	0.10%	0.28	0.56	0.40%	0.64	0.06	0.90%	0.05	0.87	4.10%	0.18	0.39
P9M3	0.60%	0.06	0.17	0.90%	0.11	0.25	1.60%	0.08	0.17	6.60%	**0.03**	0.08	0.10%	0.28	0.56	0.20%	0.13	0.07	0.90%	0.75	0.87	3.70%	0.05	0.12
P9M4	0.90%	**0.00**	**0.01**	1.40%	**0.00**	**0.00**	2.40%	**0.00**	**0.00**	6.50%	**0.04**	0.11	0.90%	**0.00**	**0.01**	1.00%	0.05	0.07	1.20%	0.54	0.26	4.70%	0.66	0.79
P9M5	0.50%	0.16	0.37	0.80%	0.22	0.44	1.50%	0.14	0.27	6.60%	**0.03**	0.08	0.10%	0.28	0.56	0.20%	0.13	0.07	0.90%	0.75	0.87	4.40%	0.37	0.67
P9M6	0.40%	0.38	0.67	0.80%	0.22	0.44	1.60%	0.08	0.17	6.70%	**0.02**	0.06	0.10%	0.28	0.56	0.10%	**0.03**	0.07	0.90%	0.75	0.87	3.70%	0.05	0.12
P9M7	1.00%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	2.60%	**0.00**	**0.00**	7.30%	**0.00**	**0.01**	1.10%	**0.00**	**0.00**	1.40%	**0.00**	0.07	2.00%	**0.01**	**0.01**	6.70%	**0.02**	0.05

(continued on next page)

Table 4 (*continued*)

	N/T 0.25%	p_{UC}	p_{CC}	N/T 0.50%	p_{UC}	p_{CC}	N/T 1%	p_{UC}	p_{CC}	N/T 5%	p_{UC}	p_{CC}	N/T 95%	p_{UC}	p_{CC}	N/T 99%	p_{UC}	p_{CC}	N/T 99.5%	p_{UC}	p_{CC}	N/T 99.75%	p_{UC}	p_{CC}
P9M8	0.20%	0.74	0.94	0.50%	1.00	0.98	1.10%	0.75	0.84	6.30%	0.07	0.17	0.30%	0.76	0.95	0.90%	0.11	0.06	1.30%	0.36	0.56	5.60%	0.39	0.69
P9M9	0.30%	0.76	0.95	0.60%	0.66	0.88	1.30%	0.36	0.56	6.50%	**0.04**	0.11	0.10%	0.28	0.56	0.70%	0.40	0.07	1.20%	0.54	0.71	5.30%	0.67	0.90
P10M1	1.00%	**0.00**	**0.00**	1.60%	**0.00**	**0.00**	2.20%	**0.00**	**0.00**	6.70%	**0.02**	0.06	0.50%	0.16	0.37	0.70%	0.40	0.67	1.30%	0.36	0.56	5.10%	0.88	0.96
P10M2	0.50%	0.16	0.37	0.60%	0.66	0.88	1.20%	0.54	0.26	5.70%	0.32	0.60	0.30%	0.76	0.95	0.50%	1.00	0.06	0.70%	0.32	0.57	4.70%	0.66	0.07
P10M3	0.50%	0.16	0.37	0.60%	0.66	0.88	1.20%	0.54	0.26	5.80%	0.26	0.49	0.30%	0.76	0.95	0.50%	1.00	0.06	0.80%	0.51	0.75	4.70%	0.66	0.07
P10M4	0.70%	**0.02**	0.06	1.20%	**0.01**	**0.01**	2.10%	**0.00**	**0.01**	6.90%	**0.01**	**0.03**	0.50%	0.16	0.37	0.50%	1.00	0.07	1.20%	0.54	0.71	4.90%	0.88	0.60
P10M5	0.40%	0.38	0.67	0.60%	0.66	0.88	0.80%	0.51	0.75	5.90%	0.20	0.43	0.30%	0.76	0.95	0.50%	1.00	0.06	0.60%	0.17	0.38	4.70%	0.66	0.07
P10M6	0.50%	0.16	0.37	0.60%	0.66	0.88	1.20%	0.54	0.26	5.80%	0.26	0.49	0.30%	0.76	0.95	0.30%	0.33	0.06	0.70%	0.31	0.57	4.70%	0.66	0.07
P10M7	0.90%	**0.00**	**0.01**	1.20%	**0.01**	**0.03**	2.50%	**0.00**	**0.00**	8.10%	**0.00**	**0.00**	1.10%	**0.00**	**0.00**	1.60%	**0.00**	0.08	2.10%	**0.00**	**0.01**	7.60%	0.00	**0.00**
P10M8	0.00%	**0.03**	0.08	0.50%	1.00	0.98	0.80%	0.51	0.75	5.50%	0.47	0.77	0.50%	0.16	0.37	0.60%	0.66	0.06	1.20%	0.54	0.71	5.90%	0.20	0.01
P10M9	0.00%	**0.03**	0.08	0.50%	1.00	0.98	0.80%	0.51	0.75	5.40%	0.57	0.85	0.50%	0.16	0.37	0.70%	0.40	0.05	1.10%	0.75	0.84	5.60%	0.39	0.09

Table 5
Asymmetric loss functions (14) and SPA tests: 1st–5th portfolios

LOSS	**0.25%**	**0.50%**	**1%**	**5%**	**0.25%**	**0.50%**	**1%**	**5%**	**SPA**	**0.25%**	**0.50%**	**1%**	**5%**	**0.25%**	**0.50%**	**1%**	**5%**
P1M1	0.7223	1.1616	1.9615	6.2138	0.6374	1.0496	1.7239	5.6260	M.1	**0.016**	**0.006**	**0.003**	**0.028**	**0.04**	**0.024**	**0.024**	0.481
P1M2	0.4927	0.8991	1.6568	6.0100	**0.4890**	0.8958	1.5798	5.6061	M.2	0.915	0.586	0.257	0.141	0.862	0.777	0.822	0.34
P1M3	0.5115	0.9447	1.7218	6.0836	0.5096	0.8951	1.5702	5.5661	M.3	0.378	0.093	**0.047**	**0.021**	**0.000**	0.804	0.824	0.505
P1M4	0.7024	1.1511	1.9537	6.2371	0.6192	1.0326	1.7109	5.6358	M.4	**0.011**	**0.003**	**0.002**	**0.013**	0.082	**0.042**	0.079	0.217
P1M5	0.4926	0.8937	1.6417	6.0134	0.5026	0.8936	1.5747	5.6096	M.5	0.976	0.544	0.302	0.175	**0.000**	0.802	0.820	0.381
P1M6	0.4925	0.9153	1.6908	6.0869	0.5287	0.8922	**1.5665**	**5.5618**	M.6	0.915	0.353	0.071	0.060	**0.000**	0.686	0.815	0.995
P1M7	0.6472	1.0917	1.8885	6.2539	0.7889	1.2034	1.8885	5.7919	M.7	0.053	0.050	**0.038**	**0.003**	**0.007**	**0.008**	**0.006**	**0.002**
P1M8	0.5075	0.8940	**1.5879**	**5.9767**	0.5228	0.9046	1.6294	5.6484	M.8	0.118	0.267	0.906	0.959	0.390	0.345	0.243	0.149
P1M9	**0.4916**	**0.8765**	1.6283	6.0443	0.4969	**0.8839**	1.5878	5.5898	M.9	0.981	1.000	0.366	0.095	0.699	0.982	0.813	0.605
P2M1	0.4303	0.7683	1.3479	5.1677	0.8107	1.2097	1.8706	5.6689	M.1	0.307	0.328	0.324	0.428	0.065	0.075	0.222	0.722
P2M2	0.4046	0.7089	1.3037	5.1615	0.6804	1.0966	1.7824	5.6555	M.2	**0.000**	0.275	0.941	0.294	0.319	0.656	0.51	0.892
P2M3	**0.3984**	**0.7029**	1.3041	5.1780	**0.6604**	**1.0867**	1.7669	**5.6512**	M.3	0.994	0.998	0.913	0.218	0.974	1.000	0.800	0.870
P2M4	0.4212	0.7640	1.3453	5.1842	0.7894	1.2050	1.8656	5.6660	M.4	0.744	0.395	0.433	0.032	0.087	0.078	0.281	0.884
P2M5	0.4073	0.7192	**1.3019**	5.1760	0.6696	1.0982	1.7740	5.6671	M.5	0.156	**0.001**	0.992	0.163	0.265	0.357	0.691	0.416
P2M6	0.4034	0.7087	1.3071	5.1880	0.6646	1.0938	**1.7633**	5.6678	M.6	0.695	0.478	0.730	0.169	0.603	0.331	0.837	0.163
P2M7	0.4342	0.7787	1.3749	5.2662	0.9629	1.3607	2.0172	5.7788	M.7	0.082	0.098	**0.033**	**0.001**	**0.010**	**0.009**	**0.011**	0.053
P2M8	0.4393	0.7721	1.3254	**5.1376**	0.7175	1.1143	1.8262	5.7166	M.8	**0.000**	**0.000**	0.280	0.920	0.349	0.646	0.397	0.226
P2M9	0.4287	0.7562	1.3205	5.1615	0.7109	1.1174	1.8258	5.7070	M.9	**0.000**	**0.000**	0.385	0.319	0.38	0.649	0.389	0.268
P3M1	0.5151	0.8730	1.5034	5.1700	0.4402	0.8067	1.4386	5.2595	M.1	0.214	0.074	0.071	0.503	0.223	0.244	0.315	0.198
P3M2	0.4382	0.7910	1.4019	**5.1329**	0.3988	**0.7220**	1.3738	**5.1925**	M.2	0.469	0.888	0.689	0.981	0.679	0.897	0.976	0.940
P3M3	0.4485	0.7998	1.4146	5.1353	0.4185	0.7590	1.3902	5.2041	M.3	0.466	0.676	0.530	0.938	**0.000**	**0.008**	0.482	0.180
P3M4	0.5084	0.8597	1.5085	5.1818	0.4344	0.7994	1.4334	5.2557	M.4	0.262	0.116	0.072	0.368	0.305	0.323	0.402	0.319
P3M5	0.4284	**0.7855**	**1.3934**	5.1338	0.4155	0.7407	**1.3734**	5.1928	M.5	0.839	0.992	0.998	0.935	**0.000**	**0.000**	0.887	0.972
P3M6	0.4298	0.7942	1.4065	5.1457	0.4339	0.7661	1.3949	5.1941	M.6	0.980	0.723	0.678	0.206	**0.000**	**0.000**	0.290	0.884
P3M7	0.4830	0.8207	1.4681	5.1917	0.5799	0.9449	1.6143	5.3441	M.7	0.318	0.507	0.170	0.199	**0.025**	**0.024**	**0.009**	0.025
P3M8	0.4330	0.7930	1.4093	5.1369	**0.3943**	0.7800	1.4277	5.2290	M.8	0.115	0.589	0.449	0.636	0.896	0.330	0.171	0.292
P3M9	**0.4231**	0.7904	1.4004	5.1373	0.3996	0.7663	1.4010	5.2144	M.9	0.999	0.861	0.788	0.863	0.731	0.466	0.687	0.677
P4M1	0.5039	0.8687	1.5060	**4.9842**	0.8837	1.2236	1.7860	**5.1306**	M.1	0.134	0.069	0.055	0.895	0.079	0.247	0.512	0.909
P4M2	0.4243	0.7747	1.4347	5.0192	0.7598	1.1843	1.7985	5.2469	M.2	0.553	0.53	0.071	0.165	0.129	0.112	0.436	0.085
P4M3	0.4310	0.7855	1.4661	5.0431	0.7164	1.1399	1.7908	5.2257	M.3	0.386	0.511	0.044	0.074	0.651	0.513	0.881	0.135
P4M4	0.4934	0.8680	1.5116	4.9975	0.8845	1.2261	**1.7829**	5.1360	M.4	0.176	0.063	0.051	0.179	0.064	0.283	0.946	0.490
P4M5	0.4144	0.7774	1.4314	5.0309	0.7375	1.1550	1.7881	5.2555	M.5	0.411	0.558	0.018	0.147	0.049	0.228	0.968	0.075
P4M6	0.4086	0.7765	1.4501	5.0597	**0.7079**	**1.1281**	1.7908	5.2343	M.6	0.993	0.670	0.020	0.039	0.875	0.915	0.692	0.141
P4M7	0.5240	0.9196	1.5752	5.0721	1.0137	1.3647	1.9650	5.3033	M.7	0.072	**0.003**	**0.002**	0.018	**0.011**	**0.008**	**0.004**	**0.010**
P4M8	**0.4063**	**0.7482**	**1.3705**	5.0025	0.8526	1.2538	1.8577	5.3547	M.8	0.761	0.877	0.838	0.643	0.077	0.144	0.191	**0.017**
P4M9	0.4100	0.7597	1.3790	5.0298	0.8306	1.2390	1.8361	5.3123	M.9	0.821	0.560	0.252	0.216	0.081	0.123	0.265	**0.033**
P5M1	1.0656	1.5936	2.4240	7.1341	0.8157	1.2786	2.0923	6.8045	M.1	**0.013**	**0.022**	**0.047**	**0.014**	0.097	0.114	0.229	0.442
P5M2	0.6988	1.2606	2.1437	6.9422	0.7208	**1.1810**	1.9905	6.7665	M.2	0.110	0.099	0.449	0.158	0.984	0.993	0.776	0.115
P5M3	0.6941	1.2962	2.1814	6.9700	**0.7200**	1.1887	1.9913	6.7354	M.3	0.145	0.058	0.081	0.051	0.915	0.521	0.565	0.614
P5M4	1.0401	1.5567	2.4034	7.1668	0.8027	1.2648	2.0790	6.8139	M.4	**0.012**	**0.024**	**0.032**	**0.003**	0.266	0.287	0.256	0.255
P5M5	0.6608	1.2219	2.1320	6.9592	0.7251	1.1834	**1.9846**	6.7703	M.5	0.411	0.176	0.468	0.095	0.788	0.901	0.998	0.163
P5M6	0.6475	1.2377	2.1479	6.9802	0.7280	1.1921	1.9878	**6.7319**	M.6	0.726	0.154	0.397	0.059	0.534	0.505	0.648	0.979
P5M7	0.9774	1.5086	2.3794	7.2616	1.0734	1.6146	2.4905	7.1181	M.7	**0.021**	**0.032**	0.056	**0.000**	**0.002**	**0.000**	**0.000**	**0.001**
P5M8	0.6358	**1.1317**	**2.0866**	**6.9043**	0.7584	1.2620	2.0629	6.9233	M.8	0.575	0.951	0.900	0.943	0.272	0.114	0.128	**0.007**
P5M9	**0.6316**	1.1718	2.1113	6.9363	0.7310	1.2220	2.0268	6.8597	M.9	0.954	0.200	0.410	0.203	0.796	0.439	0.513	0.055

the Data Generating Process (DGP). We found that, when the true copula is represented by the normal copula, there is marginal skewness in the data and symmetric marginals are used, the estimated correlations are negatively biased, and the bias increases when moving from the Student's t to the normal distribution, reaching values as high as 27% of the true correlations. Besides, we found that the bias almost doubles if negative correlations are considered, as compared to the case for positive correlations. When the true dependence function is represented by the t copula, the choice of

Table 6
Asymmetric loss functions (14) and SPA tests: 6th–10th portfolios

LOSS	0.25%	0.50%	1%	5%	0.25%	0.50%	1%	5%	SPA	0.25%	0.50%	1%	5%	0.25%	0.50%	1%	5%
P6M1	1.1366	1.7290	2.6792	8.0874	1.0944	1.6373	2.5408	7.6675	M.1	**0.010**	**0.003**	**0.003**	**0.01**	0.164	0.16	0.178	0.286
P6M2	0.6775	1.2231	2.2715	7.8308	0.9540	1.5040	2.4201	**7.5295**	M.2	0.616	0.057	**0.025**	0.404	0.573	0.195	0.497	0.94
P6M3	0.6918	1.2475	2.3144	7.8571	0.9519	1.4929	2.4251	7.5323	M.3	0.252	0.059	**0.012**	0.130	0.595	0.315	0.324	0.777
P6M4	1.1166	1.7205	2.6949	8.1196	1.0863	1.6273	2.5322	7.6778	M.4	**0.013**	**0.003**	**0.007**	**0.005**	0.184	0.211	0.342	0.230
P6M5	0.6669	1.1830	2.2371	7.8436	**0.9359**	**1.4698**	**2.4073**	7.5313	M.5	0.793	0.557	**0.042**	0.189	0.923	0.997	0.997	0.926
P6M6	0.6734	1.2191	2.2842	7.8766	0.9382	1.4873	2.4212	7.5328	M.6	0.713	0.092	0.025	0.045	0.734	0.170	0.404	0.894
P6M7	1.2359	1.8773	2.8897	8.3879	1.4000	1.9713	2.9047	8.1426	M.7	**0.005**	**0.000**	**0.000**	**0.000**	**0.005**	**0.005**	**0.002**	**0.000**
P6M8	**0.6526**	**1.1599**	**2.1399**	**7.8199**	1.0201	1.5926	2.4901	7.6361	M.8	0.776	0.834	0.949	0.957	0.277	0.195	0.350	0.143
P6M9	0.6662	1.1840	2.1900	7.8532	1.0001	1.5634	2.4740	7.6345	M.9	0.498	0.313	0.056	0.152	0.366	0.323	0.476	0.141
P7M1	0.8557	1.3811	2.2256	6.6212	0.6339	1.0828	1.8691	6.0986	M.1	**0.008**	**0.003**	**0.002**	**0.007**	0.066	**0.031**	**0.009**	0.125
P7M2	0.5912	1.0096	1.8297	6.4211	0.5269	0.9246	1.6256	5.9973	M.2	0.299	0.396	**0.034**	0.259	0.007	0.430	0.722	0.267
P7M3	0.5978	1.0262	1.8414	6.4295	0.5168	**0.8963**	**1.6143**	**5.9535**	M.3	0.198	0.115	**0.034**	0.403	0.590	0.991	0.999	0.789
P7M4	0.8244	1.3625	2.2282	6.6499	0.6189	1.0672	1.8682	6.1060	M.4	**0.008**	**0.006**	**0.001**	**0.007**	0.069	**0.023**	**0.005**	0.097
P7M5	0.5620	0.9920	1.7857	6.4394	0.5301	0.9218	1.6356	5.9959	M.5	0.823	0.819	0.214	0.087	0.130	0.191	0.242	0.397
P7M6	0.5789	1.0039	1.8012	6.4338	0.5273	0.9079	1.6170	5.9545	M.6	0.398	0.443	0.171	0.304	0.435	0.317	0.562	0.912
P7M7	0.7629	1.3053	2.1846	6.7023	0.8778	1.3782	2.1576	6.2408	M.7	**0.043**	**0.007**	**0.001**	**0.003**	**0.003**	**0.001**	**0.001**	**0.027**
P7M8	**0.5470**	**0.9795**	**1.7326**	**6.3982**	0.5302	0.9493	1.6856	6.0714	M.8	0.786	0.768	0.955	0.923	0.231	0.164	0.086	0.115
P7M9	0.5500	0.9851	1.7569	6.4134	**0.4948**	0.9200	1.6441	6.0494	M.9	0.822	0.695	0.255	0.47	1.000	0.688	0.587	0.104
P8M1	**0.3185**	0.6064	1.1324	4.2107	0.4576	0.7816	1.3793	**4.6894**	M.1	0.702	0.297	0.369	0.324	0.399	0.144	0.488	0.899
P8M2	0.3373	0.6221	1.1396	4.2154	0.4548	0.7790	1.3635	4.7131	M.2	**0.000**	**0.047**	**0.016**	0.449	0.865	0.854	0.381	0.285
P8M3	0.3346	0.6184	1.1410	4.2145	0.4549	0.7822	1.3675	4.7205	M.3	0.052	**0.026**	0.279	0.510	0.721	0.442	0.108	0.156
P8M4	0.3187	0.5980	1.1266	4.2055	0.4515	**0.7727**	1.3737	4.6901	M.4	0.783	0.880	0.716	0.656	0.939	0.988	0.615	0.884
P8M5	0.3422	0.6275	1.1339	4.2094	0.4516	0.7763	**1.3495**	4.7058	M.5	**0.000**	**0.002**	0.178	0.832	0.956	0.939	0.985	0.746
P8M6	0.3358	0.6184	1.1321	4.2101	**0.4511**	0.7851	1.3543	4.7120	M.6	**0.000**	**0.040**	0.388	0.613	0.708	0.182	0.604	0.534
P8M7	0.3208	**0.5948**	**1.1151**	**4.1982**	0.6237	0.9625	1.5292	4.7335	M.7	0.513	0.757	0.904	0.945	**0.010**	**0.014**	**0.041**	0.418
P8M8	0.3863	0.6669	1.1764	4.2274	0.4957	0.8546	1.4572	4.7189	M.8	**0.000**	**0.000**	**0.000**	0.213	0.162	0.059	0.119	0.656
P8M9	0.3712	0.6486	1.1590	4.2186	0.4847	0.8369	1.4438	4.7149	M.9	**0.000**	**0.000**	**0.006**	0.411	0.279	0.089	0.125	0.766
P9M1	0.5375	0.8865	1.5502	**5.2787**	0.7029	1.0908	1.7098	5.2699	M.1	**0.049**	0.110	0.117	0.912	**0.036**	0.110	0.237	0.448
P9M2	0.4355	0.8185	1.4745	5.2926	0.5676	**0.9080**	1.5924	5.2456	M.2	0.489	0.304	0.356	0.859	0.156	0.993	0.648	0.213
P9M3	0.4442	0.8189	1.4805	5.2956	0.5681	0.9179	1.5940	5.2657	M.3	0.279	0.468	0.376	0.858	0.529	0.427	0.370	0.069
P9M4	0.5187	0.8757	1.5413	5.2853	0.6805	1.0837	1.7116	5.2671	M.4	0.135	0.240	0.164	0.396	0.109	0.129	0.226	0.525
P9M5	0.4225	**0.8004**	1.4620	5.3014	0.5855	0.9217	**1.5832**	5.2333	M.5	0.932	0.993	0.958	0.554	**0.000**	**0.018**	0.977	0.771
P9M6	**0.4196**	0.8144	1.4700	5.3041	0.5834	0.9362	1.5962	5.2531	M.6	0.974	0.454	0.707	0.407	**0.000**	**0.000**	0.321	0.221
P9M7	0.5643	0.9526	1.6345	5.3626	0.8230	1.1866	1.7907	5.3030	M.7	**0.043**	**0.008**	**0.007**	**0.044**	**0.028**	0.052	0.077	0.231
P9M8	0.4274	0.8022	1.4628	5.2943	**0.5477**	0.9456	1.6081	5.2284	M.8	0.612	0.708	0.686	0.740	0.880	0.302	0.631	0.521
P9M9	0.4244	0.8060	**1.4611**	5.2937	0.5503	0.9246	1.5987	**5.2178**	M.9	0.716	0.694	0.923	0.886	0.825	0.74	0.887	1.000
P10M1	0.5748	0.9654	1.6482	5.6243	0.9933	**1.3357**	**1.9693**	**5.7744**	M.1	0.162	0.084	0.126	0.197	0.549	0.936	0.983	0.982
P10M2	0.4804	0.8794	1.5468	5.5822	0.9411	1.3438	1.9976	5.8839	M.2	0.433	0.702	0.394	0.080	0.899	0.95	0.757	0.145
P10M3	0.4819	0.8857	1.5435	5.5719	0.9385	1.3511	2.0292	5.8956	M.3	0.420	0.459	0.615	0.353	0.895	0.707	0.063	0.073
P10M4	0.5512	0.9315	1.6247	5.6266	0.9892	1.3428	1.9775	5.7810	M.4	0.246	0.277	0.149	0.172	0.625	0.393	0.493	0.636
P10M5	**0.4542**	0.8706	1.5321	5.6013	**0.9366**	1.3425	2.0122	5.8835	M.5	0.995	0.825	0.941	**0.020**	0.970	0.901	0.088	0.167
P10M6	0.4558	0.8697	**1.5311**	5.5853	0.9456	1.3523	2.0299	5.8878	M.6	0.882	0.893	0.991	0.204	0.024	0.300	0.061	0.118
P10M7	0.5612	0.9628	1.6598	5.7464	1.0626	1.4410	2.0683	5.9398	M.7	0.179	0.171	0.070	**0.012**	0.101	0.067	0.118	0.054
P10M8	0.4596	0.8580	1.5451	5.5690	0.9651	1.3460	2.0015	5.8849	M.8	**0.002**	0.711	0.558	0.238	0.694	0.816	0.686	0.258
P10M9	0.4569	**0.8569**	1.5446	**5.5497**	0.9696	1.3469	2.0082	5.8846	M.9	0.597	0.892	0.737	1.000	0.623	0.875	0.658	0.243

the marginals tends to have much stronger effects on copula parameter estimation, with biases up to 50% of the true values for the correlations and up to 380% for the t copula degrees of freedom parameter. If the dependence structure is represented by a copula which is not elliptical, e.g. the Clayton copula, the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of the marginal skewness.

We then implemented an extensive Monte Carlo study to assess the potential impact of both misspecified margins and misspecified copulas on the estimation of multivariate VaR for equally weighted portfolios. We found that, when small samples are considered and the data are leptokurtic and skewed, the overestimation/underestimation in the GARCH parameters is so large as to deliver conservative VaR estimates, even with a simple multivariate normal distribution. In general, the VaR estimates are very poor and suffer from computational problems when estimating GARCH models for small samples, as also discussed in Hwang and Valls Pereira (2006). When the sample dimension increases, the biases in the volatility parameters are much smaller, whereas those in the copula parameters remain almost unchanged or even increase. In this case, copula misspecifications do play a role for VaR computation. However, these effects depend heavily on the sign of the dependence: if it is negative, the bias can be as large as 70%, like for t copula correlations; if it is positive, the bias is much smaller (10% or less for the t copula correlations), and the effects on quantile estimation are much more limited, if not completely offset by marginal misspecifications. Therefore, this Monte Carlo evidence gives some insights into why previous empirical literature found that the influence of a misspecification in the copula is given with 20% or less of the whole estimation error for the VaR; see e.g. Ané and Kharoubi (2003) and Junker and May (2005). Finally, we performed an empirical analysis with ten trivariate portfolios, where we quantified the risk of the portfolio under different joint distribution assumptions.

An avenue for further research is performing a Monte Carlo analysis with high dimensional portfolios, where dynamic dependence and other forms of nonlinearities play an important role.

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